HYDRODYNAMICS OF PLATE COLUMNS. VI.*

A STUDY OF DYNAMIC PROPERTIES OF GAS-LIQUID MIXTURE ON A SIEVE PLATE WITHOUT DOWNCOMER

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The possibility of a study of the hydrodynamics of plates by means of a method of statistical dynamics was verified experimentally. Pressure fluctuations were observed in the space between the plates by means of an oscilloscope and DISA transducers. A computerized calculation has shown the distribution of the random process, its autocorrelation functions, integrals of ergodicity and estimates of deviations of found statistical quantities. It has been established that the examined process on the plate approximates best a normal ergodic random process under the regime of mobile froth. Greater deviations were found in the region of bubbling and oscillatory region. The obtained quantities represent a first approximation. Nevertheless, it can be expected that by extension of observation intervals to 100 s, the investigated technique will achieve sufficient accuracy for the whole range of operation of the plate.

Present approach to the study of the hydrodynamics of plates can be characterized as predominantly a quasi-static one. The subject of the experimental and theoretical study is usually a two-phase gas–liquid system, characterized by its pressure drop, porosity, liquid holdup and interfacial area. The main aim is then to find a relationsship between these quantities and operation and construction parameters of a plate, such as the flow rates of both phases, their physical properties, free cross-sectional area of the plate and diameter of openings. This approach suffices for description of some aspects of the plate hydrodynamics, but it cannot be applied to those phenomena where a random change is their immediate cause (*e.g.* the flow of liquid through a plate without downcomer). The experimental measurement of such quantities, primarily the total pressure drop, is not very exacting from the point of view of the experimental technique of measurement of pressure and pressure differences, since averaging both in time and space takes place. Some of the recent papers are characterized by the tendency to view a plate as a system with distributed parameters[1-4],

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which in experimental technique calls for the measurement of local values of pressure and velocities in the gas–liquid mixture. This approach seems to yield a more general description of a plate, which in turn requires a smaller number of necessary empirical parameters, and these have a definite physical interpretation. On the basis of our own results and those from the literature, we believe that a general description of the operation of a plate necessitates not a quasi-static approach, but rather an approach regarding the plate as a dynamic system with random fluctuation of quantities. It is not surprising, because the turbulent regime in the system on the plate predeterminates itself the possibility of considering the process as a random one. The plate can be regarded as a system with an inlet and outlet and its experimental and theoretical investigation can be tackled by means of the statistical dynamics of the control systems. These methods need an experimental technique of measurement of instantaneous values of quantities, in our case particularly that of pressure.

For instance, the existence of pressure fluctuations has been observed earlier, partly in connection with macro-oscillations of froth on plates[5,6]. A qualitative deterministic model, involving the effect of pressure fluctuations[7] has been worked out. Similar character has the model due to Hinze[8]. As can be seen, the deterministic approach does not suit to the character of obtained experimental data and therefore does not enable a quantitative evaluation. It is surprising that the application of statistical dynamics to the bubbling and two-phase gas–liquid flow has not been realized to date, while a whole series of its applications to fluidization[9–11] is known.

The aim of this work is to verify the aplicability of the selected method for the purpose of a further study of the hydrodynamics of plates. Later we intend to interpret the obtained results together with a theoretical model of a random process on the plate.

THEORETICAL

Let us assume that the random process – the pressure fluctuations in the space between the plates – can be regarded as stationary, normal and ergodic with respect to the expected value, variance, and correlation function. The theory of these processes is relatively elaborate[12–15] and if the experimental data confirm adequacy of the assumptions made, the first necessary precondition of a random process model is ensured. In description of these processes, within the limits so called correlation theory[13–15], we shall make do with numerical characteristics such as the expected value, variance, autocorrelation function and spectral density of a random process. For the expected value of a process $X(t)$, denoted by m_X, we can write:

$$\lim_{T\to\infty} M\left[\left\{\frac{1}{T}\int_{t_0}^{t_0+T} X(t)\,\mathrm{d}t - m_X\right\}^2\right] = 0\,. \tag{1}$$

For the autocorrelation function $R_{XX}(\tau)$ (sometimes also the autocorrelation function of pulsation of a random process[14,15])

$$\lim_{T\to\infty} M\left[\left\{\frac{1}{T}\int_{t_0}^{t_0+T}[X(t) - m_X]\,[X(t+\tau) - m_X]\,dt - R_{XX}(\tau)\right\}^2\right] = 0\,. \quad (2)$$

For the variance D_{XX}

$$\lim_{T\to\infty} M\left[\left\{\frac{1}{T}\int_{t_0}^{t_0+T}[X(t) - m_X]^2\,dt - D_{XX}\right\}^2\right] = 0\,. \quad (3)$$

For the spectral density $G_{XX}(\omega)$, a Fourier transform of an even function $R_{XX}(\tau)$

$$G_{XX}(\omega) = \frac{1}{\pi}\int_0^{\infty} R_{XX}(\tau)\cos\omega\tau\,d\tau\,. \quad (4)$$

It can be shown that the conditions of ergodicity (*1*) and (*3*) can be rewritten into the form:

$$\lim_{T\to\infty}\int_0^T (1 - \tau/T)\,R_{XX}(\tau)\,d\tau = 0\,, \quad (1b)$$

$$\lim_{T\to\infty}\int_0^T (1 - \tau/T)\,R_{XX}^2(\tau)\,d\tau = 0\,. \quad (3b)$$

In experimental observation during a finite time interval we obtain statistical characteristics only, which can serve as the estimates of corresponding quantities. A random process in a finite interval T is usually studied by means of its values in equidistant time intervals of the magnitude T/N. The error of the obtained estimates m_X^*, R_{XX}^*, G_{XX}^{**}, is markedly affected both by the length of the time interval and by the density of sampling. For these estimates we can write:

$$m_X^* = 1/T\int_{t_0}^{t_0+T} X(t)\,dt\,, \quad (5)$$

$$R_{XX}^{**}(\tau) = 1/(T-\tau)\int_{t_0}^{t_0+T-\tau}[X(t) - m_X^*]\,[X(t+\tau) - m_X^*]\,dt\,, \quad (6)$$

$$D_{XX}^{**} = (\sigma_{XX}^{**})^2 = 1/T\int_{t_0}^{t_0+T} X^2(t)\,dt - m_X^{*2}\,, \quad (7)$$

$$G_{XX}(\omega) = 1/\pi\int_0^{\infty} R_{XX}^{**}(\tau)\cos\omega\tau\,d\tau\,. \quad (8)$$

Working with equidistant intervals in practice, integration can be replaced by summation.

$$m_X^* = 1/N \sum_{k=1}^{N} X(t)_k \,, \tag{5a}$$

where $t_k = t_0 + T/N\,(k - 1)\; k = 1, 2, \ldots N$,

$$R_{XX}^{**}\left(\frac{T}{N}\,n\right) = \frac{T}{N - n} \sum_{k=1}^{N-n} X(t_k)\, X(t_{k+n}) - m_X^{*2} \,, \tag{6a}$$

$$D_{XX}^{**} = 1/N \sum_{k=1}^{N} X^2(t_k) - m_X^{*2} \,, \tag{7a}$$

$$G_{XX}^{**}(\omega) = \frac{T}{2\pi N} R_{XX}^{**}(0) + \frac{T}{\pi N} \sum_{k=1}^{\infty} R_{XX}^{**}\left(\frac{T}{N}\,k\right) \cos\left(\omega\,\frac{T}{N}\,k\right). \tag{8a}$$

Both the autocorrelation function and the spectral density and their estimates can be expressed in normalized form, *e.g.*:

$$r_{XX}^{**}(\tau) = R_{XX}^{**}(\tau)/D_{XX}^{**} \,, \tag{9}$$

$$g_{XX}^{**}(\tau) = G_{XX}^{**}(\tau)/D_{XX}^{**} \,. \tag{10}$$

Further it can be derived[13], that for the variance of the estimates of the expected value of the autocorrelation function and its variance, following is true:

$$s^2(m_X^*) = 2/T \int_0^T [1 - \tau/T]\, R_{XX}^{**}(\tau)\, \mathrm{d}\tau \tag{11}$$

or

$$s^2(m_X^*) = 1/N\, R_{XX}^{**}(0) + 2/N \sum_{l=1}^{N-1} R_{XX}^{**}\left(l\,\frac{T}{N}\right) - 2/N^2 \sum_{l=1}^{N-1} l R_{XX}\left(l\,\frac{T}{N}\right), \tag{11a}$$

$$s^2(D_{XX}^{**}) = 4/T \int_0^T [1 - \tau_1/T]\, R_{XX}^{2**}(\tau_1)\, \mathrm{d}\tau_1 \,. \tag{12}$$

Comparing Eqs (*11*) and (*12*) with (*1b*) and (*3b*) we find that the variance of both sample characteristics of the ergodic function converge to zero with time increasing to infinity. The convergence of $|R_{XX}(\tau)|$ to zero for $\tau \to \infty$ is sometimes given[13] as a practical criterion of ergodicity. The so far mentioned quantities are based on data of one realization of the random process. The accuracy and reliability of obtained characteristics is increased if we use data from several realizations of the random process. The expressions necessary for their calculation for this case have been published for example in the book of Livsic and Pugacev[13].

For normal random processes, the statistical quantities m_X^* and D_{XX}^{**} obtained in sufficiently long time intervals can also be considered as having the normal distribution of probability and consequently to consider the quantities

$$(m_X^* - m_X)/s(m_X^*) \quad \text{and} \quad (D_{XX}^{**} - D_{XX})/s(D_{XX}^{**})$$

as those with the Student distribution of probability. After a simple algebraic manipulation we arrive at expressions for the estimates of the relative deviation $\Delta(m_X^*)$ and $\Delta(\sigma_{XX}^{**})$ with the confidence coefficient $1 - \alpha$:

$$\Delta(m_X^*) = t_\alpha\, s(m_X^*)/m_X^* \tag{13}$$

and

$$\Delta(\sigma_{XX}^{**}) = \Delta(D_{XX}^{**})/2 = t_\alpha\, s(D_{XX}^{**})/2D_{XX}^{**}\,.$$

For sufficiently long time intervals we consider a critical value t_α for an infinite number of degrees of freedom.

If the autocorrelation function is known analytically, the time intervals necessary to achieve a certain accuracy of the estimate of the characteristics of a random process can be determined by calculation. In our case, where the random process is studied without prior knowledge of the course of the autocorrelation function, which, as we shall see from the following, has markedly different character for different conditions, the estimates of time intervals based on the estimate of the autocorrelation function R_{XX}^{**} represent only a guide for further experimental work. At the same time, they are a criterion of the reliability of the experimental data for further processing.

EXPERIMENTAL

The experiments were performed on a hydraulic model of a plate column 120 mm in diameter vented into the atmosphere. The equipment has been described in detail in the preceding paper[2]. Radial gas blower giving 0·05 m^3/min at 2860 r.p.m. draws the air through the plate, hydraulic section, separator of liquid carry-over and measuring line with a Venturi tube and control valve with opening into the atmosphere. The liquid circulates and it is temperature controlled between the liquid supply tank and the overflow vessel which is adjustable on a scale. The overflow vessel is connected with the center of the plate and its slight elevation above the level of the plate ensures a certain small intake of liquid for given conditions. This experimental set-up is a continuation of previous works[1,2], eliminating considerable amount of disturbing effects and the obtained experimental quantities are more accurate than those, obtained on a common through-flow plate. Liquid seepage was measured by collecting into a calibrated vessel placed just beneath the plate. The reproducibility was better than 2%. The liquid holdup was determined by measuring its volume at simultaneous stopping of the gas flow and the liquid feed from the overflow vessel.

In agreement with the preceding paper, the reproducibility was found better than 3%. The flow rate of gas was measured from the pressure difference on the Venturi tube, supplied by a ZPA pressure difference transmitter to one of the two channels of a pneumatic ZPA recorder.[18] This difference was measured simultaneously on a compensation micro-manometer MKI. Corrections for the barometric pressure, absolute pressure above the plate, temperature and humidity of the gas were taken into consideration in the calculation. The pressure probe for the measurement of the pressure above the froth, mounted in the wall of the column and shielded by a cap preventing the penetration of the entrained liquid, is connected with the ZPA transmitter. The signal of this transmitter is registered by the second channel of the recorder. Following experimental arrangement (Fig. 1) was used to measure the pressure fluctuations in the space between the plates. The pressure probe was located above the liquid level in the overflow vessel, connected by a 150 mm long tubing 2 mm in diameter with a Disa 51 DO2 low pressure transducer. According to the data provided by the manufacturer, the pressure fluctuations with the maximum amplitude ± 1000 N/m^2 are transformed in the transducer into mechanic deformations of a steel electrode 0·1 mm thick forming a membrane. The changes of capacity of this element, linked in an additional circuit 51 C 02, modulate high frequency voltage 4·4—5·6 MHz generated by a fully transistored oscillator 51 E 02. The oscillator is connected with a frequency detector and DC amplifier 51 B 02. The changes in capacity of the order 10 pF are thus transformed into the changes of voltage of then order 1 V. The 51 B 02 unit is made as a plug-in module of a universal indicator 51 B 00 with a stabilized voltage source and a two-beam oscilloscope. Symmetric output of the amplifier is supplied onto the picture tube of the oscilloscope and further, if necessary, to a registration camera 51 C 04-09 with changeable recording speed. The lower limit of the transmitted frequencies is 0 Hz, the upper limit is given primarily by the onset of the resonance frequency of longitudinal oscillations of the medium in the supply

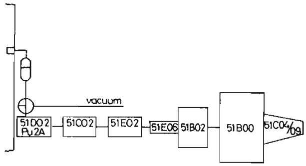

FIG. 1

Block Diagram of Pickup and Recording Devices for Measurement of Pressure Fluctuations

For explanation see Experimental.

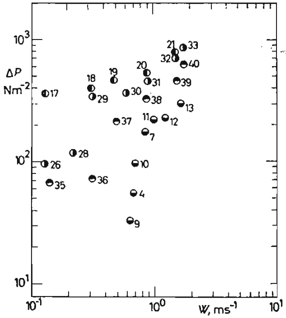

FIG. 2

Typical Dependence of Total Pressure Drop on Gas Velocity in Free Cross-section, of Plate, for Plates I—IV

⊖ Plate I, ◐ Plate II, ◑ Plate III, ● Plate IV.

channel. This frequency is higher than 1 kHz for water and a channel 200 mm long according to the data provided by the manufacturer[16]. The deviation of the amplitude and the phase shift are negligible. The calibration of the system was carried out by static pressure from the vacuum tank. A total of 40 experiments in a wide range of operating conditions, from low gas flow rates to the oscillatory regime of froth were performed with the set of selected sieve plates without downcomers investigated earlier. Their construction parameters are summarized in Table I. A whole series of experiments under the same operating conditions was carried out, while only those, whose photographic record was more clearly readable were selected for evaluation. 26 experiments were evaluated altogether. Fig. 2 demonstrates the region covered by experiments. Experimental runs number 15, 16, 34, 24, 25, measured at extremely low gas flow rates are not shown in Fig. 2. A photographic record of pressure fluctuations was obtained in every experimental run together with the calibration by 4–5 static pressures. The record was then projected on a milimeter net and read-off. Part of the records were evaluated on an Oscar apparatus by Benson and Lehner. The density of reading was taken as a fivefold of the maximum frequency[13] found, *i.e.* 48 Hz. The number of values taken from individual records was 1 600 to 2800. The size of the data set was limited primarily by the evaluation technique.

RESULTS

The calculation of the empirical probability density, the distribution function, mean m_P^*, variance D_{PP}^{**}, scewness γ_1 and excess γ_2 on the one hand, and the successive calculation of the autocorrelation function $R_{PP}^{**}(\tau)$, estimates $s^2(m_P^*)$ and $s^2(D_{PP}^{**})$ on the other hand, was performed for obtained sets of experimental data. The calculation was carried out on an Elliot computer using a program cited in the paper

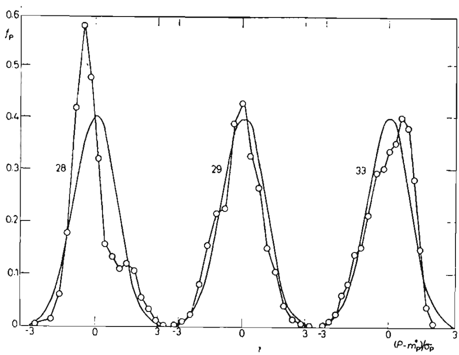

FIG. 3

Comparison of Empirical Probability Density with That of the Normal Distribution $N(m_P^*, \sigma_{PP}^{**})$, for Runs 28, 29, 33 of Koza and coworkers[12]. On the basis of the calculated quantities it can be judged to what extent the examined random process approximates the normal one. Table II compiles all information relevant to this question. Common tests of normality such as Kolmogorov's or Pearson's goodness-of-fit tests for the distribution of a random variable cannot be applied to the random processes, as there is no direct equivalent of the size of the sample[17]. Modified tests due to Svesnikov[17] are considerably more complex. We shall suppose that for the purpose of this work, a comparison of the empirical distribution functions with the normal distribution function in probabilistic coordinates, or eventually, a comparison of the empirical probability density with the normal probability density is adequate. The comparison in probabilistic coordinates was carried out for all experiments and its results are partly evident from the last but one column of Table II, comprising the maximum difference found between the empirical and theoretical distribution function. It can be concluded that more conspicuous deviations from the normal distribution are found primarily in experiments under the regime of bubbling at velocities close to the loading point, and furher in the region of the developed oscillatory regime. This is evidenced in Fig. 3 for a plate with openings 3 mm in diameter from comparison of the probability density functions in mentioned operating regions, *i.e.* 28 from the region of bubbling and 33 from the oscillatory region and for the experiment 29 from the region of mobile froth. Corresponding charts of pressure fluctuations for these experiments are shown in Fig. 4. It can be said, for the time being, that a prevailing frequency can be detected in individual records (for the experiment 28 it is the frequency of gas bubbles popping, for the experiment 29 the frequency of rotation of the blower, for the experiment 33 the frequency of macrooscillations of the froth on the plate). Accordingly, the observed process can be thought of as a process on a carrier frequency. The comparison is only qualitative cosidering that we are dealing with the realizations of the random process having different duration. This circumstance affects also, as it will be seen from the following, the accuracy of the

TABLE I

Construction Characteristics of Plates

Type	Oppenings		Free area %	Thickness of plate, mm
	number, n	diameter, mm		
I	11	10	9·19	1
II	291	2	8·21	3
III	134	3	8·53	3
IV	57	5	10·26	3

TABLE II

Comparison of Empirical Distribution of Pressure Fluctuations with the Normal Distribution

Run No	γ_1	γ_2	σ/δ	$\lvert F - F_N(m_P^*, \sigma_{PP}^{**})\rvert_{max}$	Comparison in probabilistic coordinates
			Plate I		
4	0·661	−0·096	1·254	0·0814	−
7	−0·051	−0·061	1·254	0·0225	−
9	0·062	−0·122	1·244	0·0494	+
11	0·232	−0·501	1·215	0·0584	−
12	−0·108	0·540	1·280	0·0173	+
13	0·135	0·294	1·279	0·0316	−
			Plate II		
15	0·452	0·475	1·265	0·0751	−
16	−0·080	−0·567	1·214	0·0490	−
17	−0·026	0·801	1·242	0·0251	+
18	−0·140	−0·057	1·239	0·0317	+
19	−0·103	0·362	1·285	0·0390	+
21	−0·215	−0·152	1·243	0·0476	+
			Plate III		
24	−0·049	−0·218	1·237	0·0239	+
25	−0·106	−0·058	1·245	0·0213	+
26	0·137	0·061	1·255	0·0252	+
28	0·876	0·239	1·276	0·1385	−
29	−0·051	−0·117	1·252	0·0302	+
30	−0·084	0·198	1·251	0·0210	+
31	−0·015	0·173	1·266	0·0245	+
32	−0·481	0·657	1·288	0·0265	−
33	−0·484	−0·273	1·222	0·0414	−
			Plate IV		
34	−0·244	1·134	1·266	0·0164	−
35	−1·471	16·109	1·320	0·0361	+
36	−0·085	0·008	1·265	0·0250	+
37	−0·087	0·315	1·271	0·0288	+
39	−0·582	3·690	1·265	0·0277	−
40	−0·188	−0·011	1·249	0·0282	−

determination of statistical variables m_P^*, σ_{PP}^{**}, which can affect the comparison with the normal distribution $N(m_P^*, \sigma_{PP}^{**})$.

No conclusions can be drawn from the difference in deviations from normality for different types of plates summarized in Table II. We shall therefore assume in the following that the pressure fluctuations on all investigated types of plates under the regime of mobile froth approximate best the normal random process.

Let us judge now the adequacy of the assumption of the stationary and ergodic process. The condition of convergence of $R_{XX}(\tau)$ to zero, for $\tau \to \infty$ and for long intervals of observation T, can be replaced, for finite intervals, by the condition of practical incorrelability, *i.e.* $|r_{XX}^{**}(\tau)| \leqq m$, for $T \geqq T_k$, where m is a small number. Livsic gives $m = 0{\cdot}05$ (ref.[13]). In our case, where the autocorrelation functions correspond to the random process on the carrier frequency (see Fig. 5 for plate III), the convergence may be sometimes slow and the computation for $m = 0{\cdot}05$ would require an excessive amount of computer time. Extrapolation may be considerably subjective. Therefore, in cases when convergence was faster we resorted to the calculation for $m = 0{\cdot}1$. For the rest of the data, primarily from the region of bubbling and oscillatory regime we used extrapolation according to the empirically calculated

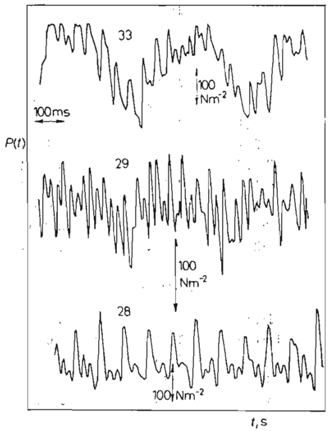

FIG. 4

Chart of Pressure Fluctuations for Plate III and Runs 28 (bubbling regime), 29 (regime of mobile froth), 33 (oscillatory regime)

Straightlines under individual curves indicate 100 N m^{-2} pressure and 100 ms time.

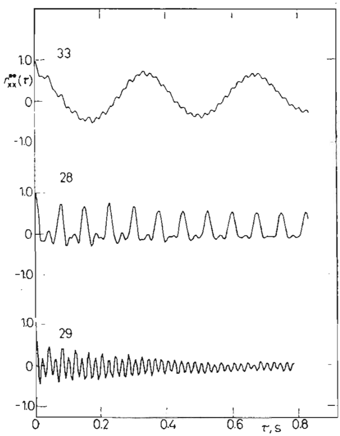

FIG. 5

Course of Empirical Normalized Autocorrelation Function for Runs under Different Regimes Length of evaluated record is 8·1 s for Run 28, 8·2 s for Run 29 and 8·3 s for Run 33.

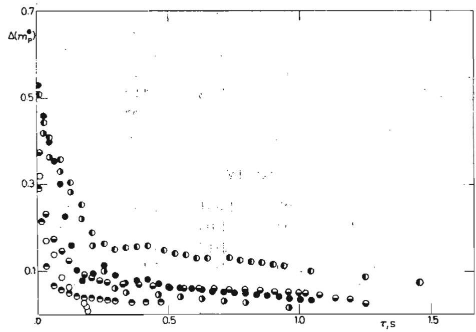

FIG. 6

Course of Estimate of Relative Deviation of Statistical Mean of Pressure Fluctuations m_P^* in Dependence on Length of Observation Interval T, for Runs under Different Regimes on Plate II

◐ Run 15, ● Run 16, ○ Run 18, ⊖ Run 19, ◑ Run 21.

TABLE III

Information about Ergodicity of Pressure Fluctuations
Time in seconds.

Run No	T_1	T_2	T_3	T_4	T_5
			Plate I		
4	0·82	0·05	2·87	0·10	27
7	0·15	0·08	0·66	0·11	8
9	0·88	0·04	1·77	0·03	33
11[a]	0·41	0·04	4·37	0·04	5
12[a]	0·25	0·04	1·54	0·04	2
13[a]	0·12	0·08	0·78	0·09	3
			Plate II		
15	3·13	0·71	4·93	1·08	1 000
16	12·10	0·14	9·03	0·15	1 000
17[a]	0·32	0·08	2·12	0·08	174
18[a]	0·76	0·08	1·99	0·08	23
19[a]	0·59	0·16	0·60	0·16	9
21	6·02	0·24	2·06	0·25	10
			Plate III		
25	0·99	0·14	4·54	0·12	270
26	0·62	0·07	1·36	0·10	13
28	12·70	4·10	4·01	0·29	1 000
29[a]	0·49	0·08	4·15	0·10	10
30[a]	0·95	0·10	1·15	0·08	58
31[a]	0·90	0·13	1·06	0·130	1 000
32	0·71	0·20	3·81	0·002	400
33	Inaccurate extrapolation — not evaluated				
			Plate IV		
34	0·70	0·03	15·81	0·03	1 000
35	3·36	0·08	1·09	0·03	1 000
36	0·46	0·03	1·15	0·06	12
37[a]	0·95	0·09	1·14	0·10	33
39	15·50	0·17	9·15	0·10	1 000
40	5·09	0·26	4·22	0·25	800

[a] Experiments under the regime of mobile froth.

best curve. The obtained times are a measure of the speed of convergence of $R_{XX}^{**}(\tau)$. This approach is suitable since we are dealing with measurements in short time intervals serving primarily as a first approximation. The found times T_1 are summarized in Table III. From this data it can be concluded that the correlability is minimum, *i.e.* the autocorrelation function converges fastest under the regime of mobile froth. In experiments from bubbling and oscillatory regimes, the convergence is considerably slower. The calculated autocorrelation functions from short time intervals indicate that even in these regimes the process can be considered as stationary and ergodic.

These facts are further supplemented by the values of integrals according to Eqs (*11*) and (*12*). Their values converge to zero with variable speed indicated by the values T_2, T_3 which are necessary to obtain the normalized values of $s^2(m_P^*)/D_{PP}^{**} = 0{\cdot}05$, respectively $s^2(D_{PP}^{**})/(D_{PP}^{**})^2 = 0{\cdot}05$ in Table III. The T_2 times were all determined by calculation, times T_3, particularly from the bubbling and oscillatory regimes, by extrapolation. These experiments show as well that the correlability of the pressure fluctuations for the regime of mobile froth is the least.

At the same time, the estimates of the relative deviations of both statistical variables were calculated in dependence on time from Eq. (*13*). These estimates were calculated for 95% confidence coefficient. A typical course of found dependences for a set of experimental runs on the plate II is shown in Fig. 6 and 7. The calculated courses were further utilized to extrapolate the values of times (T_4, T_5) corresponding to 10% relative deviation of both quantities with 95% probability. Even these values de-

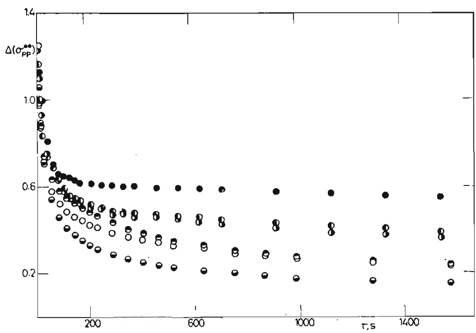

FIG. 7

Course of Estimate of Relative Deviation of Standard Deviation σ_{PP}^{**} in Dependence on Length of Observation Interval T for runs 15—21

The same graphical symbols as in Fig. 6.

monstrate more or less the fact that to achieve the same relative deviation under the regime of mobile froth, times shorter by as much as several orders are necessary. The dependence of m_p^* on the pressure $\bar{P}$, obtained from a long-term record on the pneumatic recorder (20–30 min), is plotted for comparison in Fig. 8. Considering that the dependence may be affected by the error of calibration of the transducers and the recorder, the agreement is good.

It is stressed that the times calculated by extrapolation are very inaccurate and their values should be taken as a first orientation. This is particularly true about T_5. In experiments under the regime of mobile froth the times found are comparable with experimental ones.

Under the bubbling and oscillatory regimes, the estimates are affected also by the fact that the observation times proper were, in some cases, only a fraction of those estimated by extrapolation. For further experimental work under these regimes it will be apparently necessary to increase the order of the interval of observation from 10^1 to 10^2 s, if particularly σ_{PP}^{**} is to be measured with sufficient accuracy. In accord

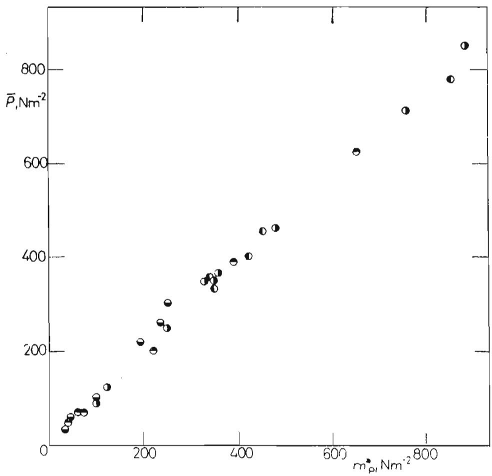

FIG. 8

Comparison of Calculated Statistical Mean m_p^* Obtained in Observation Intervals of Length 7–12 Seconds with Average Values of Long-term Measurement 20–40 Minute

The same graphical symbols as in Fig. 2.

with it, the experimental technique of measurement of random quantities has to be changed, in order that large sets of data of the order 10^4 can be managed.

LIST OF SYMBOLS

D	variance of random function
F	distribution function
$G(\omega)$	spectral density of random function
M	operator of expected value
N	number, symbol of normal distribution
$P(t)$	symbol of pressure fluctuations
$R(\tau)$	autocorrelation function of ergodic random process for time difference τ
$X(t)$	random process in time
T	interval of observation
f	probability density
m	expected value (mean)
$r(\tau)$	normalized autocorrelation function
s^2	variance
t	time
t_α	critical value of the Student distribution
δ	mean deviation
Δ	relative deviation
γ_1	scewness
γ_2	excess
σ	standard deviation
τ	time difference
ω	frequency
*, **	statistical quantity

Subscripts

X, XX, P, PP	characteristics of random processes $X(t)$ and $P(t)$ respectively
k	counting subscript

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